Mastech and Protek Measurement Uncertainties

The Mastech and Protek meter have uncertainties for each scale. These uncertainties are quoted as a % of the reading plus a number of ‘digits’.
The % of the reading is clear enough, but what is a digit ? A digital meter divides its full scale range into a number of equal width bins; that number of bins is 2 raised to its number of **bits**. Consider the Mastech on its 2 *V* range. This range covers voltages from – 2 *V* to + 2 *V*, and divides it into about 40,000 bins or 20,000 covering 0-2 *V*. On the two volt range, a Mastech bin or digit is $^{2\,V}/_{20{,}000}$ or 0.0001 *V*. With the Mastech, the range is manually selected and a digit equals the maximum positive reading divided by about 20,000. **The meter is designed such that *a digit* can be considered to be *one least significant digit* as displayed on the base 10 (decimal) scale being read (the reading resolution).**

NOTE: If the Hold button has been pressed on the Mastech, the meter appears to be frozen. Press the Hold button again to restore normal operation.

VOLTAGE: Mastech Meter set to 2 *V* and reading 1.5273 *V*.

The uncertainty spec for the 2 *V* range: ± 0.1% of rdg ± 3 digits
This translates to: $\pm\ 0.001 * (1.5273) \pm (\,^{2\,V}/_{20{,}000}) * 3 = \pm\ 1.83 \times 10^{-3}\ V$.

The value to report is (1.5273 ± 0.00183) *V* = (1.527 ± 0.002) *V*.

CAPACITANCE: Mastech Meter set to 2 μ*F* and reading 1.5273 μ*F*

The uncertainty spec for the 2 μ*F* range: ± 4.0% of rdg ± 20 digits
This translates to: $\pm\ 0.04 * (1.5273) \pm (^{2\,\mu F}/_{20{,}000}) * 20$

$$=\pm 0.06109 \quad \pm 2.00 \times 10^{-3}\ \mu F = \pm\ 0.06309\ \ \mu F.$$

Note that $(^{2\,\mu F}/_{20{,}000})$ equals 0.1 *nF* the resolution for the 2 μ*F* scale as listed in Table 4.8 Capacitance in the Mastech manual. The tabulated resolution values are the ***digit uncertainties***.

The value to report is (1.5273 ± 0.06309) μ*F* = (1.53 ± 0.06) μ*F*.

Using the Mastech Uncertainty Tables.

Uncertainty = % of rdg + (# of digits * scale resolution)

The Protek is a different kind of animal ! It auto-ranges and its bin size is the full positive range divided by about 4000 ($4096 = 10^{12}$). The range limits for the Protek are of the form 4×10^{x} (whereas they are of the form 2×10^{x} for the Mastech).

VOLTAGE: Protek Meter in the 4 V range* and reading 2.527 V.

The uncertainty spec for the 4 V range: ± 0.5% of rdg ± 2 digits
This translates to: ± 0.005 * (2.527) ± ($^{4\,V}/_{4000}$) * 2 =
= (0.01264 ± 0.002) V = 0.015 V.

The value to report is (2.527 ± 0.015) V.

Note that the value ($^{4\,V}/_{4000}$) is the *resolution* for the 4 V range and that the uncertainty is correctly estimated using the relation as a *digit*:

Uncertainty = % of rdg + (# of digits * scale resolution)
= ± 0.005 * (2.527) ± 2 *($^{4\,V}/_{4000}$)

* *Beware: the Protek changes ranges as the value measured changes. Be sure you know the range that is being used. It should be the most sensitive range that contains the measured value. That is: a voltage 3.91V is measured using the 4 V scale while 4.27 V is measured using the 40 V scale. Two digits on the 40 V scale is 0.02 V rather than the trivial 0.002 V that equates to two digits on the 4 V scale.*

Propagating Uncertainties: Examples for capacitors in series and parallel.

Series: Each measurement yields an expected (bar) value with its associated uncertainty. $C_1 \to \overline{C_1} \pm \Delta C_1$ and $C_2 \to \overline{C_2} \pm \Delta C_2$

The next step is to propagate the uncertainties in the measured values to find the uncertainty in a derived or computed quantity. The main rules are:

<u>RULE U1</u>: You **add** absolute uncertainties when you add or subtract values.

<u>RULE U2</u>: You **add** relative uncertainties when you multiply or divide quantities.

In the expression $C_1 \to \overline{C_1} \pm \Delta C_1$, ΔC_1 is an absolute uncertainty. Re-expressed, $C_1 \to \overline{C_1} \pm \Delta C_1 \to \overline{C_1}\left[1 \pm {}^{\Delta C_1}\!/_{C_1}\right]$ where ${}^{\Delta C_1}\!/_{C_1}$ is the relative (fractional) uncertainty.

Predicting the capacitance for adding in series.

$$C_s = \frac{C_1 C_2}{C_1 + C_2} \to \frac{\left(\overline{C_1} \pm \Delta C_1\right)\left(\overline{C_2} \pm \Delta C_2\right)}{\left(\overline{C_1} \pm \Delta C_1\right) + \left(\overline{C_2} \pm \Delta C_2\right)}$$

Apply U2 to the numerator and U1 to the denominator.

$$C_s \pm \Delta C_s \to \frac{\left(\overline{C_1} \pm \Delta C_1\right)\left(\overline{C_2} \pm \Delta C_2\right)}{\left(\overline{C_1} \pm \Delta C_1\right) + \left(\overline{C_2} \pm \Delta C_2\right)} = \frac{\left(\overline{C_1}\ \overline{C_2}\right)\left(1 \pm \left[\frac{\Delta C_1}{C_1} + \frac{\Delta C_2}{C_2}\right]\right)}{\left(\overline{C_1} + \overline{C_2} \pm \left[\Delta C_1 + \Delta C_2\right]\right)}$$

Convert the uncertainty in the denominator to the relative form and apply U2 to the division.

$$C_s \pm \Delta C_s = \left[\frac{\overline{C_1}\ \overline{C_2}}{\overline{C_1}+\overline{C_2}}\right]\left[1 \pm \left[\frac{\Delta C_1}{C_1}+\frac{\Delta C_2}{C_2}+\frac{\Delta C_1+\Delta C_2}{C_1+C_2}\right]\right]$$

Do measured values and predicted values agree ? Ask yourself the question: Do the ranges of values measured ± uncertainty and predicted ± uncertainty overlap ? If they overlap comfortably, you report that the value predicted agrees with the measured value to within the accuracy of the experiment. If the value ranges barely overlap, you report that the results suggest the formula predicts the value of the series combination, but that the result is more uncertain. If the value ranges barely fail to overlap, you report that the results may suggest the formula predicts the value of the series combination, but that the result is not supported by the current set of measurements. If the value ranges fail to overlap, you report that the results suggest the formula does not predict the value of the series combination to the accuracy of the experiment. **In all cases**, you include the measured and predicted values with their uncertainties in the final summary report.

Addition of Capacitors in Parallel:

Derive the uncertainty propagation formula for capacitors in parallel.

$$C_p = C_1 + C_2 \rightarrow \overline{C_p} \pm \Delta C_p = ???$$

Discuss carefully the degree to which your measurements support the rule for the addition of capacitors in parallel.

Exercise 1: Consider the case that $\tau = {}^{1}/_{RC}$. The resistance read is (39.2±0.2)Ω, and the capacitance read is (0.025 ± 0.001) *mF*. Compute τ with its uncertainty.

Exercise 2: The 200 Ω resistance scale is the most sensitive for the Mastech, and it has a resolution of 0.01Ω and uncertainty specs of 0.5% rdg ± 10 digits. Two resistors are tested leading to measured values of 39.2 Ω and 21.9 Ω. Compute the uncertainties for each measured value and report the results in the $\overline{R} \pm \Delta R$ form. Compute the values expected for the series and parallel combinations of these resistors propagating the uncertainties to yield predicted values $\overline{R}_S \pm \Delta R_S$ and $\overline{R_{||}} \pm \Delta R_{||}$. The resistance of the parallel combination is measured to be 14.1 Ω. What is the uncertainty for this result value? Do the predicted and measured values of the parallel combination agree ? The measured value for the series combination is 60.2 Ω. What is the uncertainty for this result value? Do the predicted and measured values of the series combination agree ? It is noted that shorting the meter test leads gives a meter reading of 0.3 Ω. Might this explain the apparent disagreement ?